exchange between N_2 and DCl molecules but smaller than the VT relaxation length. We also see that even for moderate $D_c = 5000h_*$ (1 m) the value of N_e in a mixing laser lasing in the fundamental transitions is 25% higher than in an ordinary GDL and may reach 62 J/g for F_k = 300. With increasing D_c this difference becomes even larger. At the same time for small $D_c = 500h_*$ the value of N_e is the same in both types of laser. This is because for a small D_c only a small fraction of the energy stored in the N_2 vibrations is converted into the energy of coherent radiation. In a mixing GDL operating on transitions with m = 2 a value $N_e \geqslant 10$ J/g can be obtained even for $D_c \geqslant 2000h_*$.

In summary, our analysis shows that a gasdynamic laser with mixing of N_2 and DCl can be a very efficient device for obtaining radiation with λ = 5-7 μm (fundamental-frequency transitions) and with λ = 2.5-2.8 μm (harmonics). The specific radiant energy may reach 70 and 20 J/g, respectively, in systems of moderate size.

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NUMERICAL ANALYSIS OF A PLASMA JET IN A MAGNETIC FIELD

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UDC 533.95

The behavior of supersonic plasma jets in an external magnetic field is of interest for problems involving the use of plasma accelerators of various types in space technology. The interplay of the jet and the magnetic field must be known if such problems are to be solved. The complexity of the pertinent experiments stimulates the use of computer simulation to obtain fuller theoretical concepts about the nature of the behavior of plasma jets. Comparatively few theoretical studies on plasma formations in a magnetic field have dealt directly with supersonic plasma jets. We note that in [1] Savel'ev used the one-temperature MHD approximation for a numerical analysis of a plane plasma jet bounded in the transverse direction.

In the work reported here, within the framework of the two-temperature MHD model we have considered the behavior of a highly underexpanded (nearly a vacuum type) supersonic plasma jet with a superimposed magnetic field, taking the induced magnetic field into account. We have studied the effect of the magnetic field on the geometry of the jet boundary, the nature of the flow, the distribution of parameters in the jet, and the perturbation of the external magnetic field by the electric currents of the jet.

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1. The Physical Model. We consider an axisymmetric, highly underexpanded jet of completely ionized plasma flowing into a nonconducting medium with a finite, but fairly low pressure. The plasma is assumed to have a finite conductivity, which is a function of the electron temperature. The external magnetic field is assumed to be directed along the jet axis so that it does not change the position of the axis in space and the jet retains its axial symmetry. The MHD approximation is used to describe the jet.

In a zero magnetic field the approximation of a continuous medium in the jets flowing into a highly rarefied space is generally incorrect since the mean free path of particles at a distance from the source becomes comparable with the characteristic jet dimensions. If the magnetic field is fairly strong, however, the mean free path of ions in the direction perpendicular to the magnetic vector is determined by the Larmor radius, which may be smaller than the characteristic transverse dimension of the jet. The magnetic field also causes the longitudinal characteristic dimension of the jet to increase. The hydrodynamic description thus becomes applicable to the entire jet: in the initial part because of the high collision rate and in the subsequent part because of the high Larmor frequency and the large characteristic longitudinal dimension of the jet. Gus'kov et al. [2] and Cowling [3] point out that a qualitatively correct result can be obtained by using the continuous medium model, even when this model is formally inapplicable.

The flow pattern is described by using the following model assumption, simplifying the two-fluid system of equations of plasma dynamics obtained in [4].

1. The ionic gas is considered to be nondissipative and electron → ion energy transfer is ignored in view of the high mass ratio and the relative low plasma density.
2. The polytropic equation of state is used instead of the energy equation for electrons. This is because in many cases the electronic thermal diffusivity is fairly high so that the electronic gas may be considered to be virtually isothermal or nearly so. The problems with using this approximation was discussed in greater detail in [5]. Instead of the polytrope equation, however, we can use other model or empirical relations since the variation of these relations does not alter the qualitative picture of the interaction of the plasma jet with the magnetic field.
3. The Hall currents are ignored. This approximation excludes the current loop in the longitudinal section of the jet. Inclusion of the Hall currents would cause an azimuthal ponderomotive force to appear and induce various parts of the jet to rotate about its axis in different directions. The effect of the Hall constant on the flare angle of the jet is determined by the centrifugal force due to the azimuthal rotation of the plasma. In a real case inclusion of the finite but large mean free path of the particles along the magnetic field of the interaction of parts of the plasma turning in different directions results in a low total azimuthal velocity and, hence, a small centrifugal force.
4. The electrical currents are assumed not to be carried out of the plasma source, i.e., the current density through the initial cross section is zero.

With the above assumptions, if the radius of the initial cross section of the jet and the velocity and density on the axis of the jet are taken to be the main dimensional quantities, the dynamic equation of the plasma [4] can be written in dimensionless form

$$\operatorname{div}(\rho\mathbf{u}) = 0; \tag{1.1}$$

$$\rho(\mathbf{u}\nabla)\mathbf{u} + \nabla p + \nabla p_e = \mathbf{j}\times\mathbf{H}; \tag{1.2}$$

$$p = \text{const}\,\rho^{\gamma}; \tag{1.3}$$

$$\sigma = \text{const}\,T_e^{3/2}; \tag{1.4}$$

$$p_e = \rho T_e; \tag{1.5}$$

$$T_e = \text{const}\,\rho^{\gamma_e - 1}; \tag{1.6}$$

$$\mathbf{j} = \sigma\mathbf{u}\times\mathbf{H}, \tag{1.7}$$

where p and p_e are the ionic and electronic pressure; γ is the adiabatic exponent for the ionic gas; γ_e is the polytropic exponent for the electronic gas; $\mathbf{u} = (u_r,\ 0,\ u_z)$. $\mathbf{H} = (H_r^0 + H_r^i,\ 0,\ H_z^0 + H_z^i)$ are the plasma-velocity and magnetic vectors in the $(r,\ \varphi,\ z)$ coordinate system; H_r^0 and H_z^0 are the components of the applied magnetic field; and H_r^i and H_z^i are the components of the induced field. The rest of the notation is conventional.

The simplicity of Eq. (1.7) is determined by the axial symmetry of the problem and the boundary conditions. The conductivity is zero and current in the medium surrounding the plasma jet and currents do not flow through the jet boundary. When assumptions 2-4 are made the only nonzero component of the current-density vector is azimuthal.

The magnetic field induced by the electrical currents in the jet is determined by Maxwell's equations. For the stationary formulation of the problem they can be written in dimensionless form,

$$\operatorname{div} \mathbf{H}^i = 0; \tag{1.8}$$

$$\operatorname{curl} \mathbf{H}^i = 4\pi \mathbf{j}. \tag{1.9}$$

Equations (1.8) and (1.9) are valid in all space, i.e., in the jet and in the surrounding medium. The plasma and the surrounding medium have the same magnetic susceptibility, equal to one. The boundary conditions for (1.8) and (1.9) are set for infinity, where the induced magnetic field is zero.

2. Method of Solution. The system of equations (1.1)-(1.9) is solved by separately solving the dynamic equations of the plasma (1.1)-(1.7) and Maxwell's equations (1.8), (1.9), and then iterating them together until a consistent solution is obtained.

We solved Eqs. (1.1)-(1.7) by using the finite-difference march method, based on an explicit scheme of second order-of-magnitude accuracy, of the Lax—Wendrof type, and expounded in [6]. In determining the jet boundary we took the effect of the magnetic field into account; one of the equations, as in [6], was the projection of the equation of motion (1.2) onto the r axis of the cylindrical coordinate system, and the integral of the equation of motion (1.2) along the jet boundary was considered to be the second equation.

On the basis of potential theory, we write the solution of Eqs. (1.8) and (1.9) as

$$H_r^i(r, z) = \int\limits_V j_\varphi(r', \varphi', z') \frac{1}{r} \frac{\partial}{\partial r}\left(\frac{r}{B}\right) dv'; \tag{2.1}$$

$$H_z^i(r, z) = \int\limits_V j_\varphi(r', \varphi', z') \frac{\partial}{\partial z}\left(\frac{1}{B}\right) dv', \tag{2.2}$$

where B is the distance from the point (r, φ, z) (because of the axial symmetry we set $\varphi = 0$), where H_r^i and H_z^i are determined, to the element of volume dv' with coordinates (r', φ', z').

Integrals (2.1) and (2.2) are improper and the integrands have a singularity at B = 0, but they can be shown to converge uniformly. They are calculated by using the cell method [7], with second order-of-magnitude accuracy.

The integration in (2.1) and (2.2) is carried out over the volume of the jet. Since the jet generally is unbounded in the z direction, we choose some value z_k sufficiently distant from the source for the current density to be zero beyond it ($z > z_k$). This assumption is allowable in the problem under consideration since it does not change the structure of the electrical currents in the jet but merely cuts off the contribution of distant current regions to the induced magnetic field. How the contribution of the currents cut off affects the induced magnetic field is assessed by appropriate parametric calculations for various values of z_k. In the calculations the effect of the currents of the regions cut off was insignificant for the bulk of the jet.

The calculations were performed on a net with 100 points in the radial direction. The spacing in the longitudinal direction was determined from the Courant stability condition. An uneven auxiliary net of 20 points along the r coordinate axis and 100 points along the z coordinate axis was introduced to calculate the induced magnetic field. The values of the components of the magnetic vector calculated at the node of this net when solving Eqs. (1.1)-(1.7) were interpolated to the main net. Parametric calculations of the induced magnetic field [at an arbitrary point it was determined from interpolation on the auxiliary net and by solving (2.1) and (2.2)] showed that this net is applicable.

3. Results of Numerical Analyses. We considered an argon plasma jet at constant parameters in the initial cross section on the jet axis: density $n_a = 10^{14}$ cm^{-3}, ion and electron temperature ($T_i = T_e$) = 0.2 eV, velocity $u_a = 6 \cdot 10^5$ cm/sec. The external magnetic field was assumed to be uniform, directed along the z axis, and have a single component $\mathbf{H}^0 = (0, 0, H_z^0)$. The radius R_a of the initial cross section is assumed to be 10 and 50 cm, $\gamma_e = 1.1$, and

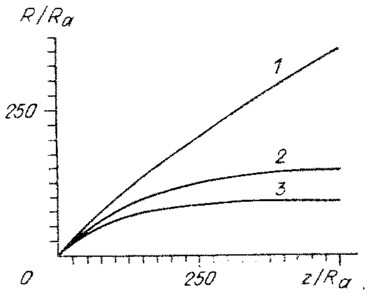

Fig. 1

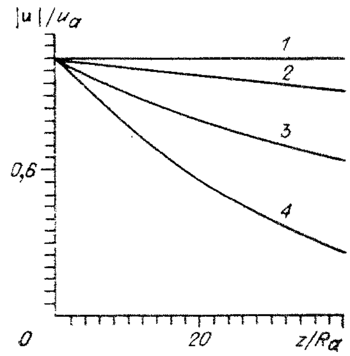

Fig. 2

γ = 1.67. We considered the case of classical conduction [4]. The plasma parameters in the initial cross section (density, two components of the velocity vector) were given by the model relations

$$\mathbf{F}=\exp\left(-\mathbf{C}r^{l}\right),\ r\in[0,1],\ l=1,2,3,\ \ldots,\ u_r=u_z\,\mathrm{const}\left(\sin\frac{r\pi}{2}\right)^{l}$$

($\mathbf{F}=[\rho,u_z]^{\tau}$, $\mathbf{C}=[c_1,c_2]^{\tau}$ are constants). The magnitude of the vector of the applied magnetic field was varied.

We studied the underexpanded jet with a 10^5 ratio of the density on the jet axis in the initial cross section to the density of the external medium. With this density ratio the jet behaves virtually as a vacuum jet.

First we consider the effect of a magnetic field on a plasma jet with R_a = 10 cm. In this case the magnetic Reynolds number $Re_m \approx 1$. As is seen from Fig. 1, where the boundary of the plasma jet is shown for various values of H_z^0 (H_z^0 = 0, 0.5, 1 Oe, lines 1-3), the magnetic field substantially changes the shape of the jet boundary. The jet begins to be compressed, to an increasing extent when the ponderomotive force is stronger (Fig. 1, curves 2, 3). At some values of the vector magnitude the applied magnetic field, e.g., for H_z^0 = 1 and 0.5 Oe or less, the ponderomotive force simply compresses the jet boundary. When H_z^0 increases to 1.5 Oe the nature of the flow in the jet changes and zones of subsonic flow form. Numerical calculations showed that with supersonic flow the jet radius has a minimum value (in the given case $R_* \approx 90$ caliber), which is reached at the limiting ponderomotive force. If the ponderomotive force exceeds the limiting value, the nature of the flow in the jet changes, and regions of subsonic flow appear. Under the given conditions this is observed for H_z^0 = 1.5 Oe and z = 40-50 caliber. As the ponderomotive force increases further, the zone of subsonic flow moves closer to the plasma source.

The mechanism by which regions of subsonic flow appear in the plasma jet in the magnetic field is due to the dissipative effect that the magnetic force has on the plasma velocity. With zero magnetic field for a jet flowing into a homogeneous external medium, the magnitude of the velocity vector at the boundary field line is constant (Fig. 2, line 1). The magnetic field hinders radial expansion of the jet, thus reducing the radial component of the velocity vector, most markedly near the jet boundary. From Fig. 2 we see that the plasma near the boundary is slowed more when the magnetic vector is larger (lines 2-4 correspond to H_z^0 = 0.5, 1, and 1.5 Oe). The gas pressure gradient causes a local redistribution of the boundary components of the velocity vector: the axial component decreases while the radial component increases, in the final account appreciably decreasing the velocity vector (Fig. 2, lines 2-4). The decrease in the velocity vector in the core of the jet is not as substantial as in the boundary region.

Another feature of the effect of the magnetic field on the plasma jet is a change in the density profile in the jet. From Fig. 3, where the density profiles are shown in the cross section z = 400 for H_z^0 = 0, 1, and 1.5 Oe (lines 1-3) we see that compression of the jet raises the plasma density in the jet and qualitatively changes its profile. Compactions due to retardation of the plasma appear in the boundary region (Fig. 3, curves 2, 3). The appearance of such compactions at the boundary between the plasma and the external medium was noted, in [8] in particular, where Colgate pointed out that they appear when the plasma is expanded in a magnetic field. The effect of the induced magnetic field on the jet with R_a = 10 cm was insignificant.

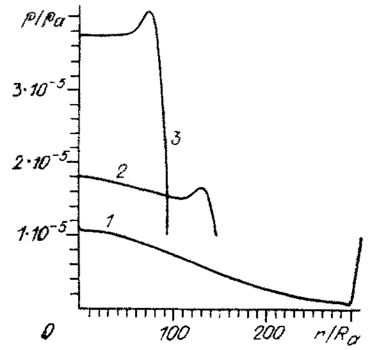

Fig. 3

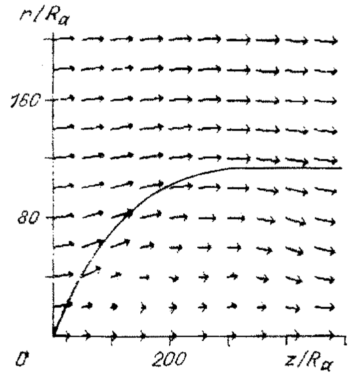

Fig. 4

When R_a increases to 50 cm (in this case $Re_m \approx 3$) the effect of the induced field becomes appreciable and the jet becomes much broader than when the induced magnetic field is ignored, while the plasma density is lower and the density peaking at the boundary is larger.

The distribution of the total magnetic field, i.e., the sum of the applied field and that induced in the jet and surrounding space at R_a = 50 cm, is illustrated in Fig. 4. The greatest perturbation of the applied field manifests itself in the plasma jet and its vicinity. This perturbation of the magnetic field is insignificant at a distance (r = 200). Three regions, each with a characteristic distribution pattern, can be distinguished for the total magnetic field:

1) the initial segment of flow, where the jet expands strongly; here the applied magnetic field is squeezed out of the jet because of the high currents, especially in the boundary region;

2) the middle (z = 150-350) axial part of the jet, when the magnetic field is very low;

3) the remote part of the jet, where the magnetic field begins to penetrate more actively into the weakly expanding jet with lower electrical currents.

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